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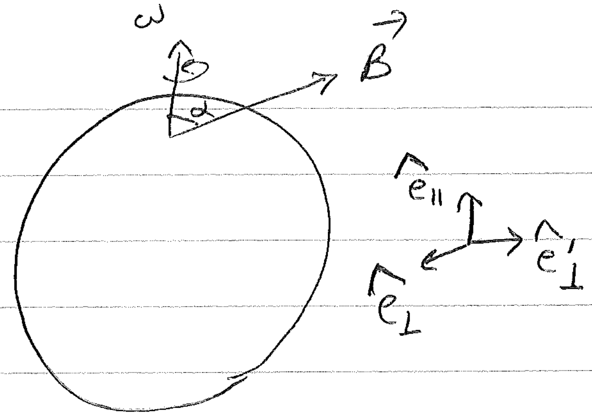
Pulsars:

As we saw, one of the possible end stages of stellar evolution could be a highly compct and dense distribution of matter in the form of a neutron star. The angular momentum will be conserved during the collapse to a large degree. If the initial angular momentum $J_i \simeq M R_i^2 \omega_i$ is non-zero, the conservation of angular momentum implies that $J_f \simeq M R_f^2 \omega_f = J_i$, thus $\omega_f = \omega_i \left(\frac{R_i}{R_f}\right)^2$. If $R_i \sim 10^5 R_f$, then $\omega_f \sim 10^{10} \omega_i$. Therefore pulsars can have a very short period of rotation.

As the core collapses to form a neutron star, its electrical conductivity becomes very high (due to high density of free electrons). The magnetic flux through the star $\Phi_M \propto BR^2$ is conserved in the limit of infinite conductivity, which implies that $B_f \sim B_i \left(\frac{R_i}{R_f}\right)^2$. If $B_i \simeq 100$ G and $R_i \sim 10^5 R_f$, we get $B_f \simeq 10^{12}$ G. Therefore pulsars can have a very strong magnetic field.

It is clear that we are led to stellar remnants threaded by a very high magnetic field in a state of rapid rotation. This will lead to emission of radiation by means of different processes. The primary probe of such structures is through the electromagnetic radiation. In general, the rotation and the magnetic field define two different axes in the pulsar (misaligned rotator). The pulsar acts as a rotating magnetic dipole that will emit electromagnetic radiation (essentially at the frequency of rotation) with a dipole pattern. To make an order of magnitude estimate, consider a rotating neutron star surrounded by vacuum. The magnetic moment $\vec{m}$ is then given by:

$$\vec{m} = \frac{1}{2} B R^3 \left(\hat{e}_{\parallel} \cos\alpha + \hat{e}_{\perp} \sin\alpha \cos\omega t + \hat{e}'_{\perp} \sin\alpha \sin\omega t \right)$$



The time varying dipole moment emits radiation at a rate;

$$\frac{dE}{dt} = -\frac{2}{3c^3}\,|\ddot{\vec{m}}|^2 = \frac{-B^2 R^6 \omega^4 \sin^2\alpha}{6c^3}$$

Note that for $\alpha = 0$ (aligned rotator) the magnetic dipole moment is constant in time, hence no dipole radiation. The energy radiated away is supplied by rotational kinetic energy $K = \frac{1}{2} I\omega^2$ ("I" being the moment of inertia). Thus;

$$\frac{dE}{dt} = I\omega\dot{\omega} = \frac{-B^2 R^6 \omega^4 \sin^2\alpha}{6c^3}$$

We can define a characteristic time scale;

$$\boxed{T \equiv -\left(\frac{\omega}{\dot{\omega}}\right)_0 = \frac{6Ic^3}{B^2 R^6 \omega_0^2 \sin^2\alpha}}$$

Where the subscript o denotes the current value. Assuming that $\omega = \omega_i$ at $t=0$, we then find;

$$\omega = \omega_i \left(1 + \frac{2\omega_i^2}{\omega_0^2} \frac{t}{T}\right)^{-\frac{1}{2}}$$

The present age of the pulsar (assuming that $\omega_0 \ll \omega_i$) is found to be;

$$t \simeq \frac{T}{2}$$

We can use these expressions to make simple estimates regarding emission from pulsars. For a spherical neutron star with $M \simeq 1.4 M_\odot$ and $R \simeq 12$ km (and $I \simeq 1.4 \times 10^{45}$ g cm^{-3}), the kinetic energy is $K \simeq 2.5 \times 10^{49}$ erg (for $P \simeq 0.03$ s). The Crab pulsar, for example, formed as a result of a supernova explosion that occured in 1054 A.D., which implies that $T \simeq 10^3$ yr. This results in $\dot{\omega} \simeq 10^{-8}\ s^{-2}$ and an energy loss rate $\simeq 6.4 \times 10^{38}$ erg s^{-1}. If $\sin\alpha \simeq 1$, the magnetic field is found to be $B \simeq 5.2 \times 10^{12}$ G.

The very strong magnetic field and stability of the period are evident from the underlined numbers.

In the simple model discussed above, we have made some approximations:

(1) The region surrounding the pulsar is assumed to be a vacuum that allows radiation to escape freely.

(2) General relativistic effects have been ignored.

(3) The electromagnetic fields have not been solved inside and outside the neutron star a matched at the surface.

In particular, regarding (1), a rotating magnetic dipole will create a strong electric field near the surface of the pulsar. It can pull out charged particles from the surface, as a result of which the pulsar can be surrounded by a plasma with high conductivity.

The problem will be that of a rotating star surrounded by a (co)rotating plasma. The net radiation emitted by the system should take into account all the processes operating in this magnetosphere of the plasma.

For realistic values of the plasma density, the dipole radiation will not be able to propagate through the plasma. The magnetosphere, however, can lead to different radiative processes. For example, the charged particles spiralling around the magnetic field lines will emit curvature radiation. Moreover, high energy photons propagating in the magnetosphere could result in electron-positron pair production. These will be accelerated by the electric field in the magnetosphere and emit radiation.

It is important to note that the plasma cannot strictly corotate with the pulsar at arbitrarily large radial distances. The reason being that the linear velocity of a particle $v = \omega r$ cannot exceed the speed of light. This leads to a maximum radius for corotation, called the radius of the light cylinder:

$$R_L \equiv \frac{c}{\omega} = 4.77 \times 10^9 \text{ cm} \left(\frac{P}{1s}\right)$$

The net effect of R_L is to reconfigure the electromagnetic fields in such a way that the magnetosphere plasma emits radiation to large distances. The resulting emission (arising primarily from the charged particles moving along the magnetic field lines that originate near the poles of the pulsar) can be detected on Earth periodically when the radiation cone sweeps past us at the rotational period of the pulsar.

The magnetosphere radiation depends crucially on the existence of a plasma around the pulsar, but is independent from the details of alignment between the magnetic field and the rotation axes. The details regarding the generation of the plasma around the pulsar can be very complicated and are ill understood. However, there are two elementary conclusions that are reasonably robust and independent of the detailed nature of the emission mechanism.

(1) Pulsars emit radiation with well defined periodicity. The period is stable.

(2) The pulsars constantly losing energy because of a variety of radiation processes. As a result, the angular velocity of pulsars decreases over time (the spin-down effect).

Because the period P can be measured with a high level of accuracy, we can estimate $\dot{P}$ and consequently test several of the model predictions. Also, the extraordinary stability of the pulsar period makes pulsars a useful probe of several other physical pheomena. For example, the radiation from a pulsar received on Earth propagates through the Inter-Stellar Medium (ISM) that contains plasma and magnetic fields. By estimating the Faraday rotation and plasma dispersion for the radiation, we can estimate the interstellar magnetic fields and electron density. In a more general case, even weak perturbations that cause variations in the period of the pulsar signal can be measured accurately and hence can be used to probe the source of perturbations. Let us discuss this in some detail now.

Pulsar Timing:

As mentioned, the pulses of radiation emitted by pulsars are remarkably stable and act as a very precise clock. The study of arrival time of the pulses from the pulsar over a long duration of time allows us to determine the period P and its time derivative $\dot{P}$ with considerable accuracy. In some cases the precision in the measurement of P is comparable with that in terrestrial atomic clocks.

The timing measurements can be used to determine the pulsar position in the sky. Because Earth orbits around Sun, the radiation from the pulsar travels different distances during different times of the year. Assuming (for simplicity) that the orbit of Earth is circular, the time delay is given by:

$t_c = A \cos(\omega t - \lambda) \cos\beta$

A: light travel time from Sun to Earth

ω: angular velocity of Earth in its orbit

λ: ecliptic longitude of the pulsar, β: ecliptic latitude of the pulsar

One has to determine and correct for various known effects:

(1) Lengthening of the pulsar period (due to spin-down effect)

modifies the sinusoidal variation in above.

(2) The rotation of Earth introduces a variable time delay (which is ~21 ms).

(3) The center-of-mass of the solar system is just outside the surface of Sun. The motion of the telescope with respect to the center-of-mass needs to be corrected for.

(4) The observed frequency varies because of the Doppler effect due to Earth's motion around Sun.

(5) The rate of the clock is affected by the depth of gravitational potential well in which the clock is situated. As Earth moves around Sun in an elliptical orbit, it samples slightly different gravitational potential due to Sun. Thus Earth-bound clock rate will show a small annual variation compared with a clock in a circular orbit.

All these effects are now precisely calculable and are routinely taken into account in timing studies.

The variation in the arrival time of the pulse due to phenomena taking place in the ISM is of importance in theoretical studies of the structure of the ISM. Another interesting application of pulsar timing is to detect gravitational waves. Observations of a large sample of pulsars spread across the celestial sphere forming a "Pulsar Timing Array" can in principle enable a positive detection of the gravitational wave background in the galaxy. The effect of gravitational waves is that they distort the spacetime as they pass through a region. This will cause a difference in the depth of gravitational potential wells for different pulsars in the sample, which changes their relative time delays. In fact, pulsar timing sets stringent bounds on a background of gravitational waves that may have been produced cosmologically.